Elucidation of Small-Angle Electron Scattering and Lassettre's Limit Theorem

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We present a simple representation of the apparent generalized oscillator strength (AGOS) which permits the demonstration of its general properties. We show that only the zero angle trajectory connects continuously the threshold energy, $E = \omega$, and $E = \infty$ corresponding to the optical oscillator strength (OOS) limit of the AGOS. Far-reaching implications include the applicability of Lassettre's limit theorem regardless of E, the normalization of measured electron differential cross sections (DCSs) and the identification of their spurious behavior. Illustrative results are presented for optically allowed transitions in H, Li and Ba.

The difficulties of measuring reliably and normalizing the electron DCSs at small scattering angles are well documented [1,2]. Even the most recent measurements of the DCS's for H [3] obtained data down to only $\theta = 7°$, and for Li, their limited availability reflects technical problems [4].

The AGOS (atomic units are used) is related to the DCS, $\frac{d\sigma}{d\Omega}$ through

$$AGOS(K, p_i) = \frac{\omega}{2}\frac{p_i}{p_f}K^2\frac{d\sigma}{d\Omega} \tag{1}$$

where the momentum transfer $\vec{K} = \vec{p_i} - \vec{p_f}$ with $\vec{p_i}$ and $\vec{p_f}$ being the electron momenta before and after the collision, respectively, The AGOS converges to the OOS at K=0 regardless of the electron energy [5].

The representation [6] $x = \frac{K^2}{2\omega}, t = \frac{\omega}{E}$ and $y = cos\theta$ transforms the expression for K^2 to $xt = 2 - t - 2\sqrt{1-t}\, y$, with $x \geq 0$ and $0 \leq t \leq 1$. In these variables the general properties of the AGOS and the applicability of the Lassettre limit theorem are readily demonstrated. As an example we use the resonance transition in Li.

The Li $2s - 2p$ transition has been selected because of the availability of measurements [1,4] and calculations [7,8] over a wide range of E and θ values. These data provide a stringent test of our approach and, vice versa. Fig. 1(a) shows the AGOS versus t curves from the data of Bray *et al.* [8] at $\theta = 0°, 1°, 3°, 5°, 10°$ and $15°$ (represented by the top curve $\theta = 0°$, down to the bottom curve $\theta = 15°$, respectively) from $t = 0.00185$ ($E = 1000$ eV) to $t = 0.185$ ($E = 10$ eV). The AGOS's of Ref. [8] are the most impressive demonstration of how they approach the OOS limit as $E \to \infty$ and θ decreases from 15° to 0°. The measured data points [1] are shown at $\theta = 3°$, 5° and 10°. The agreement with the data [8] is excellent, except at t=0.185 (E=10 eV). Note that at this energy, the AGOS's at $\theta = 0°$, 3° and 5° are quite close together.

Fig. 1(b) contrasts data from measurements [1,4] and calculations [7,8] at $\theta = 0°$ for values of E from 7 eV to 1000 eV. Since the data [8] represent the zero-angle scattering excellently, clearly the measurement [1] (crosses) require renormalization. The data of Ref. [7] (triangles) behave spuriously only at $\theta = 0°$. The new measurement at 14 eV (diamonds) [4] underestimates the Bray *et al.* forward scattering data, while at 7 eV it agrees excellently with them. The lower curve represents the data [8] at $\theta = 3°$ and the pluses the measurement [1]. We note that near 1000 eV the data at $\theta = 0°$ and 3° are clearly separated, but by $E = 7$ eV they are already indistinguishable, including the data point of Ref. [4]. This has interesting consequences for measurements.

The behavior in Fig. 1 is general and is also exhibited by the more complicated Ba and simple H atoms. We will demonstrate the applicability of Lassettre's limit theorem over the entire E, thereby resolving a problem which has remained unsolved for nearly four decades. Future work calls for the study of the limiting value for the AGOS as $E \to \omega$ and the exact functional form of the forward scattering AGOS whose utility cannot be emphasized enough.

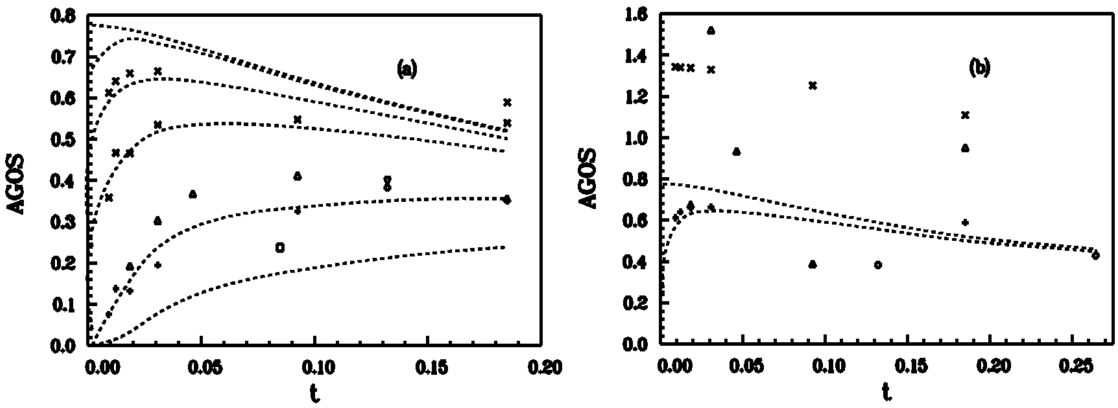

Figure 1: Comparison of the AGOS versus t for Li $2s - 2p$ from DCS measurements [1,4] and calculations [7,8]

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